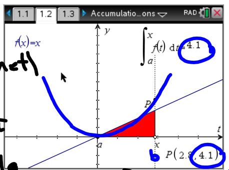
5.3 Definite Integrals and Antiderivatives

A function defined by a definite integral

$A(x)=\int_a^x f(t)\,dt = \frac{x^2}{2}$ area (net)

y-coord of $P = \int_a^x f(t)\,dt$

P-traces out a parabola



$A(x)=y=\frac{x^2}{2} = \frac{b^2}{2}$

$A'(x)=x$

inverse of derivative
antiderivative
integral

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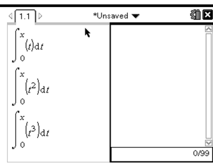
Use FTC Discovery.tns to evaluate the following integrals and look for a pattern.



exp & denominator increase by one

$\int_0^x t^n\,dt = \frac{x^{n+1}}{n+1}$

antiderivative of x^n is $\frac{x^{n+1}}{n+1}$

$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right)=x^n$

same

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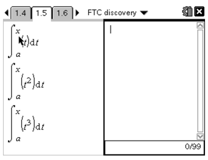
Evaluate the following integrals and look for patterns



$\int_a^x t^n\,dt = \frac{x^{n+1}}{n+1}-\frac{a^{n+1}}{n+1}$

$\int_a^b x^n\,dx = \frac{b^{n+1}}{n+1}-\frac{a^{n+1}}{n+1}$

FTC

$$\int_a^b f(x)\,dx = F(b)-F(a)$$

$F(x)$ is the antiderivative of $f(x)$

or

$F'(x)=f(x)$

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Evaluate the following definite integrals by hand

1.1 *Unsaved

$F'(x)=\sin x \quad F(x)=-\cos x$

$\int_0^\pi (\sin(x))\,dx = -\cos x\Big|_0^\pi = \overset{-(-1)}{-\cos\pi} \overset{+\,1}{-\ -\cos 0} = 2$

$\int_2^3 (x^2+x-1)\,dx = \frac{x^3}{3}+\frac{x^2}{2}-x\Big|_2^3 = \left(\frac{3^3}{3}+\frac{3^2}{2}-3\right)-\left(\frac{2^3}{3}+\frac{2^2}{2}-2\right) = \frac{47}{6}$

$\int_0^1 \left(\frac{1}{1+x^2}\right)dx = \tan^{-1}x\Big|_0^1 = \tan^{-1}1-\tan^{-1}0 = \frac{\pi}{4} - 0 = \frac{\pi}{4}$

$\tan\frac{\pi}{4}=1 \qquad \tan 0=0$

$\tan^{-1}1=\frac{\pi}{4} \qquad \tan^{-1}0=0$

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